Gamma Fragility

The Price Effects of Delta-Hedging

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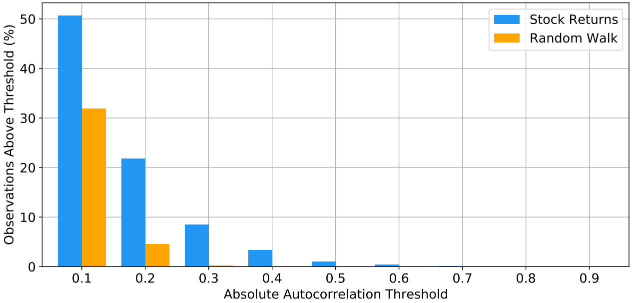
Returns Auto-Correlation

- Extensive literature exists on returns auto-correlation
- Strong evidence supporting TS and XS momentum
- Auto-correlation is present also at higher frequency, e.g. 5-minutes returns

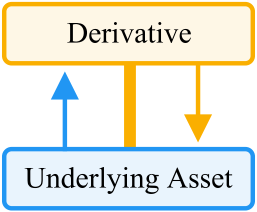


- **Question:** Are financial intermediation frictions in option/insurance market responsible these price dynamics?

Feedback Effects

- Black and Scholes assume that options are redundant, so that they can be replicated using the underlying stock
- This may not hold in presence of relevant frictions
- Unusual hedging demands in an imperfectly liquid equity market may induce feedback effects:

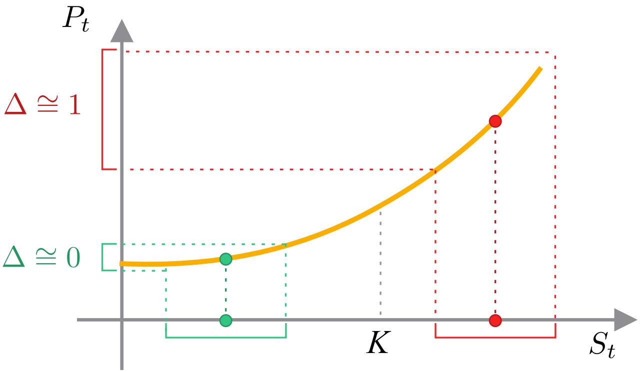
$$\text{Underlying} \Longrightarrow \text{Derivative} \Longrightarrow \text{Underlying}$$



Δ, Γ and Delta-Hedging

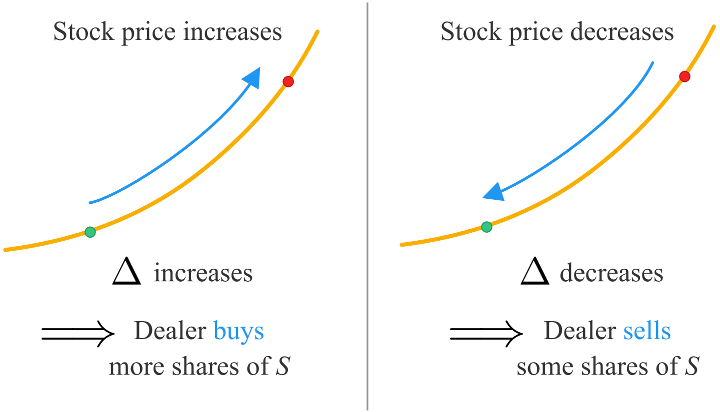
- The sensitivity is of the option price to the underlying stock price is $\Delta = \frac{\partial}{\partial S} P$

- **Delta-Hedging:** dealer buys/sells Δ of the underlying stock to hedge against price movements in the underlying



Dynamic Delta-Hedging

- Because of **convexity** , dealer has to constantly re-balance the hedging portfolio. For instance, for a call option:



- The re-balancing is proportional to the **change in Δ**, that is, to the option **Gamma** (denoted as Γ)

Positive and Negative Gamma

- Delta-hedgers can be both long or short call and put options
- When dealers are long, delta-hedging forces them to trade in the opposite direction of stock returns

Position	Terminology	Re-balancing	Effects
Dealer sells options	Dealer is short Gamma	Same direction of price moves	• Volatility increase • Positive correlation
Dealer buys options	Dealer is long Gamma	Opposite direction of price moves	• Volatility decrease • Negative correlation

- We exploit this symmetry to identify the effect of delta-hedging on stock price dynamics

Imbalance of Incentives

- In the aggregate, gamma and all other greeks are net zero
- However, not all market participants engage in delta-hedging
- Some market participants have incentives to delta-hedge
  - Option dealers and market makers
  - Hedge funds
- Others are not interested in delta-hedging
  - Active funds (protective collars or other option strategies)
  - Retail investors (leveraged bets)
  - Insurance companies (buy put options to hedge market risk)

$\Longrightarrow$ Γ-induced flow can be non-trivial

- Risk transfer in the supply chain of options markets

  Households $\Longrightarrow$ Insurances $\Longrightarrow$ Dealers $\Longrightarrow$ Hedge funds

Testable Implications

Dealers' Δ-hedging impacts stock prices

1. Stock returns volatility depends on dealers' Gamma exposure
2. The effect is more pronounced for less liquid stocks
3. Intra-day returns autocorrelation depends on dealers' Gamma
4. When dealers' Gamma is large, flash crash events are more likely

Related Literature

Derivatives affects the price of underlying securities

- Ben-David, Franzoni, and Moussawi (2018)
- Shum, Hejazi, Haryanto, and Rodier (2015)
- Ni, Pearson, Poteshman (2020)

Economic frictions, liquidity, and market fragility

- Brunnermeir and Pedersen (2009)
- Holmstrom and Tirole (1997)

The effect of institutions on asset prices

- Vayanos and Woolley (2013)
- Gorton, Hayashi, and Rouwenhorst (2013)
- Hendershott and Seasholes (2007)

Data Sources

- We use volume data from four option exchanges from the **CBOE** (C1) and the **Nasdaq** (ISE, GEMX, and PHLX)
- Signed trading volume on individual option contracts broken down by category of market participants, daily
  - Broker/Dealers $\longleftrightarrow$ $\text{VOL}^{\text{Dealers}}$
  - Proprietary trading desks $\longleftrightarrow$ $\text{VOL}^{\text{Props}}$
  - Retail Investors $\longleftrightarrow$ $\text{VOL}^{\text{Retails}}$
- Options on more than 3000 US equity stocks from January 2010 to May 2020
- Merge with OptionMetrics, CRSP and TAQ data

Dollar Gamma Exposure

- For each **option** i, we compute daily inventories held by different investor groups, cumulating signed volume

$$\mathrm{INV}_i^g(t) = \sum_{s \leq t} \mathrm{VOL}_i^g(s)$$

- For each **stock** j, the aggregate gamma of **group** g is the gamma-weighted sum of the inventory across options written on that stock

$$\Gamma_j^g(t) = \sum_{\substack{\text{options } i \\ \text{written on} \\ \text{stock } j}} \Gamma_i(t) \times \mathrm{INV}_i^g(t) \times \frac{S_j(t)}{100} \times S_j(t)$$

- This is the dollar amount to be traded by investor group g to re-hedge after a 1% movement in the price S_j of stock j

Hedgers Gamma Exposure

- We assume only Broker/Dealers and market makers employ delta-hedging, and therefore define:

$$\Gamma_j^{\text{Hedgers}}(t) \;=\; \Gamma_j^{\text{Dealers}}(t) - \Gamma_j^{\text{Retails}}(t)$$

- As a robustness check, we also test the case in which proprietary trading desks are also assumed to delta-hedge

$$\Gamma_j^{\text{Hedgers 2}}(t) \;=\; \Gamma_j^{\text{Dealers}}(t) + \Gamma_j^{\text{Props}}(t) - \Gamma_j^{\text{Retails}}(t)$$

- Our results are robust to this alternative specification

Gamma Imbalance

- Define the aggregate **Gamma Imbalance** Γ_j^{IB} on stock j as

$$\Gamma_j^{IB}(t) = \frac{\Gamma_j^{\text{Hedgers}}(t)}{ADV_j(t)}$$

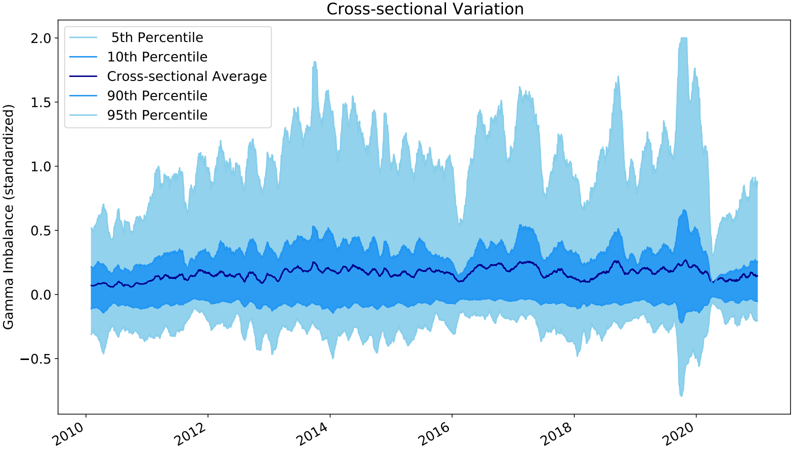
where $ADV_j(t)$ is the average daily volume for stock j as of day t, in dollars

- $\Gamma_j^{IB}(t)$ measures the amount of **price pressure** arising from dealers' re-balancing of their hedging portfolio, assuming that the stock price moves by 1 percentage point

- Our data covers most of the trading volume of options on US equity stocks, but we do not observe OTC trades

  $\Longrightarrow$ Makes it more difficult to identify the effect

Gamma Imbalance – Panel Variation



- Gamma imbalance enjoys significant time-series and cross-sectional variation

Testable Prediction

- If dealers have **negative gamma** exposure, they are forced to re-balance following stock price movements
  $\Longrightarrow$ induce positive auto-correlation in returns
  $\Longrightarrow$ volatility increases

- On the contrary, a **positive gamma** exposure forces them to trade against price movements
  $\Longrightarrow$ induce negative auto-correlation in returns
  $\Longrightarrow$ volatility decreases

- To test this, we run the following panel regression

$$|R_{j,t}| = \alpha + \beta_0 \Gamma^{IB}_{j,t-1} + \beta_1 IVOL_{j,t-1} + FE_{j,t} + e_{j,t},$$

- Prediction $\beta_0 < 0$: negative gamma leads to higher volatility

Gamma Imbalance and Volatility

	(1)	(2)	(3)	(4)
Dependent Variable	|Return|	|Return|	|Return|	|Return|
Gamma Imbalance (lag)	-114.870 ***	-75.031 ***	-125.380 ***	-63.706 ***
	(-11.296)	(-9.642)	(-15.517)	(-10.391)
Implied Volatility (lag)	81.725 ***	71.941 ***	72.504 ***	47.245 ***
	(19.477)	(7.356)	(78.553)	(20.767)
Stock Fixed Effects		Yes		Yes
Time Fixed Effects			Yes	Yes
R-squared	0.148	0.055	0.114	0.021
Observations	4,040,882	4,040,882	4,040,882	4,040,882
SEs Clustered By	Stock-Time	Stock-Time	Stock-Time	Stock-Time

- A standard deviation decrease in Γ^{IB} is associated to an increase in volatility from 63 to 115 basis points

Price Pressure Channel and Illiquidity

- Dealers re-balancing affects stock prices through a **price pressure** channel
- As a consequence, the effect of Gamma Imbalance on stock returns should be larger for less liquid stocks
- We hence rank stocks by Amihud illiquidity ratio and split the sample in two groups: more liquid and less liquid stocks, based on the median Amihud ratio estimated on the full sample
- We then run a generalized difference-in-differences regression, to identify the role played by illiquidity

Price Pressure Channel and Illiquidity

- As predicted, the effect is stronger for less liquid stocks

	(1)	(2)	(3)
Dependent Variable	|Return|	|Return|	|Return|
Low Liquidity	17.588 ***		18.407 ***
	(10.963)		(11.312)
Dollar Gamma Imbalance (lag)		-1.454	0.976
		(-1.557)	(0.840)
Dollar Gamma Imbalance (lag) × Low Liqui...			-10.955 ***
			(-8.210)
Implied Volatility (lag)	99.202 ***	103.030 ***	99.108 ***
	(37.053)	(41.908)	(36.927)
R-squared	0.523	0.522	0.523
Observations	4,040,882	4,040,882	4,040,882
SEs Clustered By	Stock-Time	Stock-Time	Stock-Time

Intra-day Auto-Correlation

- If dealers hedge during the day, this should have an impact on intra-day returns autocorrelation
- When dealers have **negative gamma** exposure
  $\Longrightarrow$ induce positive auto-correlation in returns
- When dealers have **positive gamma** exposure
  $\Longrightarrow$ induce negative auto-correlation in returns
- Using intra-day data from TAQ, we estimate the sample autocorrelation of h-minute returns for each day-stock pair, with $h = 5, 10, 20, 30, 60$
- We then run the following panel regression, for every h

$$\rho^h_{j,t} = \alpha^h + \beta^h_0 \, \Gamma^{IB}_{j,t-1} + FE_{j,t} + \varepsilon_{j,t}$$

Intra-day Auto-Correlation

	(1)	(2)	(3)	(4)	(5)
Dependent Variable	5 Min AC	10 Min AC	20 Min AC	30 Min AC	60 Min AC
Gamma Imbalance (lag)	-21.019 ***	-14.607 ***	-23.074 ***	-20.450 ***	-19.489 ***
	(-5.953)	(-3.604)	(-5.005)	(-3.936)	(-2.734)
Traded Volume	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***
	(26.012)	(19.951)	(17.136)	(20.969)	(3.840)
Implied Volatility (lag)	-0.001	-0.044	0.032	-0.001	-0.447 **
	(-0.010)	(-0.652)	(0.337)	(-0.012)	(-2.147)
Stock Fixed Effects	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes	Yes
R-squared	0.003	0.002	0.002	0.001	0.000
Observations	1,675,196	1,675,196	1,675,196	1,675,196	1,675,196
Ses Clustered By	Stock-Time	Stock-Time	Stock-Time	Stock-Time	Stock-Time

- A standard deviation decrease in Γ^{IB} is associated to an increase in autocorrelation from 14% to 21%

Identification of Flash Crashes

- We follow the **drift burst detection** methodology proposed by Christensen, Oomen, and Reno' (2018)
- The idea is to identify large price drops materializing in a short time window
- Assume prices follow a geometric Brownian motion

$$dp_t = \mu_t dt + \sigma_t dW_t$$

and define the *velocity of the market* as the test statistics

$$T_t^n = \sqrt{\frac{h_n}{K_2}} \frac{\hat{\mu}_t^n}{\hat{\sigma}_t^n}$$

where $\hat{\mu}^n$ and $\hat{\sigma}^n$ estimate the drift and diffusion of dp_t

Identified Flash Crashes

- T_t^n is estimated on minute-frequency prices from TAQ data, on 30-minutes non-overlapping windows
- Flash crashes are events with T_t^n in the lowest percentile
- The methodology identifies $11,405$ events for $2,623$ stocks
- About 1 crash every three years for each stock, on average
- Average draw-down: -2.06% (median: -1.60%)
- Define a stock-day dummy "Flash Crash" indicating that a stock experienced a flash crash on a particular day and run

$$\text{Flash Crash}(j,t) = \alpha + \beta_0 \Gamma^{IB}_{j,t-1} + FE_{j,t} + \varepsilon_{j,t}$$

Probability of Flash Crashes

- One standard deviation decrease in Γ^{IB} increases probability of flash crash from 8% to 12%

	(1)	(2)	(3)	(4)
Dependent Variable	crash	crash	crash	crash
Gamma Imbalance (lag)	-12.5750 ***	-7.9536 **	-11.0960 ***	-5.3013 **
	(-2.869)	(-2.475)	(-2.775)	(-1.978)
Implied Volatility (lag)	0.049 **	0.005	0.077 ***	0.029 ***
	(2.286)	(0.150)	(3.216)	(2.616)
Stock Fixed Effects		Yes		Yes
Time Fixed Effects			Yes	Yes
R-squared	0.000	0.000	0.000	0.000
Observations	4,569,764	4,569,764	4,569,764	4,569,764
Ses Clustered By	Stock-Time	Stock-Time	Stock-Time	Stock-Time

Magnitude of Flash Crashes

	(1)	(2)	(3)	(4)
Dependent Variable	Event Return	Event Return	Event Return	Event Return
Negative Gamma Imbalance	-211.480 ***	-24.999 ***	-20.471 ***	-18.081 ***
	(-21.619)	(-5.587)	(-4.739)	(-4.129)
Implied Volatility (lag)		-85.926 ***	-80.789 ***	-77.950 ***
		(-17.970)	(-19.810)	(-19.696)
Traded Volume			-0.000 ***	-0.000 ***
			(-9.497)	(-9.238)
Amihud Ratio				-437.650 ***
				(-6.243)
Stock Fixed Effects				
Time Fixed Effects				
R-squared	0.327	0.741	0.752	0.754
Observations	11,413	11,413	11,413	11,413
Ses Clustered By	Stock-Time	Stock-Time	Stock-Time	Stock-Time

Concluding Remarks

- We uncover a feedback effect of derivatives on the price of the underlying assets
- The aggregate gamma imbalance of option dealers is linked to volatility and autocorrelation of the underlying stocks
- The effect works through a price impact channel, and it is stronger for more illiquid stocks
- Our results can help explain excess volatility and momentum/reversal of intra-day returns
- Moreover, we document a significant impact on the probability and magnitude of flash crash events
- Red flag for regulators: increasing volume in option markets may lead to flash crash events and price spirals